DISCHARGE CALIBRATION METHODS FOR VERTICAL GATED SPILLWAY

Nguyen Cong Thanh

Summary: Discharge estimation accuracy is very important in design, operation of hydropower or irrigation project. To calculate the discharge capacity of the gated spillway, we can use the theoretical, empirical, or semi-empirical formulas. All of these equations depend on the geometry of spillway, the gate opening and the total head of the spillway. The accuracy of these equations also depends on the actual and limited conditions in each particular case. In this study, some discharge equations will be introduced and compared to data from an experimental hydraulic model of flow through the vertical gated spillway that is designed by Waterways Experiment Station (WES) shape. It indicates that the relative error of these equations was large because of the different selection of discharge coefficient in each equation. To improve the accuracy of discharge capacity, alternatively, a new formula to calculate the discharge coefficient is obtained in this field, which based on nonlinear multiple regression and ordinary least square method of recorded data. It shows that this equation agrees well with the experimental observations in given experiment conditions.

Keywords: Gated spillway; discharge equation; discharge coefficient; hydraulic experiment; multiple regression.

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1. Introduction

The spillway is one of the most important structures in hydropower or irrigation project around the world. The principal function of a spillway is to pass down the surplus water from the reservoir into the downstream river, water supply or controlling floods, etc. [1]. A spillway structure equipped with gates is often called “gated spillway”. Gated spillways are generally chosen for one of such following reasons [2]:

- To maintain reservoir at a constant water level in order to stabilize production at a hydroelectric power plant.
- To fix the volume of water for irrigation purpose.
- To control downstream flooding, or maximization of conservation storage, requires more flexible control than would be provided by a free spillway.

Obviously, accurate discharge estimation is very important from an operational, environmental and an economical point of view in irrigation or hydroelectric project. It affects the ability of safety in the activities and operation of the entire work system. Discharge prediction depends on many factors such as water upstream level, sharp-edge of the gate, the curved bed after gate, gated position on the crest spillway and head loss of flow over slot and entrance... There were some researchers worked and suggested some empirical or semi-empirical discharge formulas in gated spillway and there have been encouraging results to date. Henderson [3] obtained a discharge coefficient $C_q$ as a function of the gate opening, contraction coefficient and headwater for the case of free-orifice flow. Based on vertical gated spillway equation of Hydraulic Design Criteria [4] and experimental data, Hager [5] proposed a generalized formula to compute the discharge through vertical gate on spillway in both gated and ungated flow. Ferro [6] obtained the stage-discharge relationship for a flow simultaneously discharging over and under a sluice or a broad-crested gate by dimensional analysis and Pi-theorem. Ansar, et al [7] introduced the two discharge equations in both controlled free and submerged flow and they deduced that contraction coefficient $C$ was related to the prevailing flow conditions. The Hydrological Engineering Center (HEC) of the U.S. Army Corps of Engineers [8] in the HECRAS program uses free-orifice and free-weir equations.

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to calculate flow rate. The program considers a gated spillway to be submerged when \( h_s / H_r \geq 0.67 \), where \( H_r \) = headwater energy above the spillway. In contrast to the plethora of reduced-scale laboratory experiments on flow through gated spillways, Ansar and Chen [9] developed a generalized flow rating by using dimensional analysis and flow measurement collected mostly with an Acoustic Doppler Current Profiler at about 90 prototypes gated spillways in South Florida, U.S. Recently, Sepúlveda, et al [10] has benchmarked the performance of several calibration methods for submerged sluice gates using experimental data of a laboratory canal.

The aim of this paper is to investigate the accuracy of some discharge equations of gated spillways in free-orifice flow with the WES downstream profile and the vertical upstream face. Furthermore, a new discharge coefficient formula is obtained and compared to the experimental data. The accuracy of this formula is also compared with some existing formulas under the given laboratory conditions.

2. Flow rate formulas

2.1 Classical Theoretical Formulation

\[ H_s = z + \frac{p}{\gamma} + \frac{u^2}{2g} + \xi \frac{u^2}{2g} + \lambda \frac{L_\alpha}{D_h} \frac{u^2}{2g} \]  

(1)

in which, \( H_s \) is the total head, \( z \) is the elevation of \( dz \) above the datum plane, \( p \) is the pressure, \( g \) is the gravitational acceleration, \( L_\alpha \) and \( D_h \) are the length and hydraulic diameter of the orifice, respectively, \( \xi \) is the minor head loss's coefficient and \( \lambda \) is the Darcy friction factor between two sections. \( u \) is the velocity at streamline \( dz \). The other symbols are shown in Fig.1. After some transformations, the discharge in orifice flow, \( Q_s \), is determined by the following formula:

\[ Q_s = VA = \frac{1}{\sqrt{\alpha \left( 1 + \lambda \frac{L_\alpha}{D_h} + \xi \right)}} \sqrt{2gC_s b} \sqrt{2gZ_o} \]  

\[ Q_s = C_{0s} G_s b \sqrt{2gZ_o} \]  

(2)

where, \( C_{0s} = C_r / \sqrt{\alpha \left( 1 + \lambda \frac{L_\alpha}{D_h} + \xi \right)} \) is discharge coefficient in this task, \( C_r \) is the vertical contraction coefficient with \( h_s = G C_r \) and \( Z_o = H_s - \Pi \) is the water head effect of orifice flow. \( V \) is the average velocity at section C-C, \( G_s \) is the gate opening.

in which, \( \Pi \) is called average potential energy's outlet section, one has:

\[ \Pi = \frac{1}{G} \int \frac{z + p(z)}{\gamma} dz \]  

(3)
2.2 Typical Empirical Methods

According to Eq. (3), the vertical pressure distribution $p(z)$ and the vertical contraction coefficient $C_z$ at the cross-section C-C (see Fig. 1) must be known before calculating the discharge. However, since the contraction coefficient varies with the amount of gate opening, the shape of the gate lip, upstream water depth, gate type, and so forth, it is very difficult to know its real value for all operating conditions in practice. Additionally, the pressure distribution $p(z)$ also depends on the downstream profile of the spillway, water head $H$, and gate opening $G_0$. These mean that the application of Eq. (3) for vertical gated spillway is quite difficult due to the less accurate value measurement as well as determination. Particularly, some assumptions are often made in order to simplify this task in hydraulic practice. One of the assumptions often applies to calculate $Z_a$ value in this case. In other words, the $Z_a$ value is determined more simply; and the discharge coefficient would be found by a calibration method from the experimental data.

Generally, the discharge formula could be classified into two groups for the flow in this field. The first group, $Z_a$ value was calculated from the total energy level (TEL) to the crest or gate seat elevation of the orifice; and the second group, the value of $Z_a$ was calculated from the water upstream level to the center of the orifice, respectively. As a result, the discharge coefficient could be obtained by using regression method. Some of formulas in these groups are presented as follows:

2.2.1 Group 1

Respect to this group was some author as Hager [5], HEC-RAS [8]. In 1988, Hager [5] based on semi-empirical approach and fundamental formula in [12], proposed a formula to compute the discharge through vertical gate on standard spillway in free flow condition, written as

$$Q_s = C_{a0}L \sqrt{2gH_o} \left[ 1 - \left( \frac{Z_a}{\chi_a} \right)^{3/2} \left( \frac{1}{6} + Z_a \right)^{3/2} \right]$$

(4)

where $C_{a0}$ is the discharge coefficient of standard spillway that corresponds to design head, $Z_a = H / G_o$ and $\chi_a = H / H_o$. The range of this equation may be specified as $G_o / H_o < 4.5$ regarding the gate opening relative to the head $H$ for free flow. His equation is valid for both gated and free flow over standard spillway. Besides that, HEC-RAS [8] is the popular (freeware) software developed at the Hydrologic Engineering Centers, which is a division of the Institute for Water Resources-U.S. Army Corps of Engineers, that allows performance of one dimensional steady and unsteady flow river hydraulics calculations. The free orifice flow was conducted with condition $h_o / H_o \leq 0.67$ in this case.

$$Q_s = C_{a0}L \sqrt{2gH_o}$$

(5)

where $C_{a0}$ is fixed user-defined value (selection in the range 0.6–0.8). This approach has a less effort in the calculation of the discharge coefficients. It is also worth pointing out that this method corresponds roughly to the use of fixed discharge coefficient values.

2.2.2 Group 2

Respect to this group may include Ansar [7] and Wortman [13]. The flow equation and the restrictions on the flow in Ansar [7] are:

$$Q_s = C_{a0}L \sqrt{2g(H_o - 0.5G_o)}$$

(6)

With condition $H_o / G_o > 1.7$ and $h_o / G_o < 0.5$, $C_{a0}$ is taken constant value 0.75

Wortman [13] represented the discharge equation that used in NWS BREAK software in some case such as uncontrolled spillway, gated spillway and dam crest overflow... The free orifice flow in this case was written as follows:

$$Q_s = C_{a0}L \sqrt{2gZ_o}$$

(7)

in which $C_{a0}$ and $Z_o$ are determined by equations as follows:

$$C_{a0} = 3.9 \cdot 0.1825L \left( \frac{H_o}{H_a} \right)^{3/4}$$

(8)

$$Z_o = H_o - \max(0.5G_o, h_o)$$

(9)

Evidently, one can see that the Wortman method uses the value $C_a$ as a function of $H_o$, $H_a$, $L$. On the contrary, the constant value of $C_a$ in Ansar method that may lead to significant deviation result in comparison to Wortman method.
3. Experimental setup and procedure

3.1 Experimental setup

The experiments were carried out in a laboratory flume which consists of a steel frame with transparent Plexiglas sides, 40 cm in width, 600 cm in length and 80 cm in depth and the bottom of the flume was made of stainless steel with horizontal slope. A standard spillway of design head $H_s = 20$ cm and 34.2 cm in height ($P$) was installed into this flume. Its upstream face was a vertical gate that was made of steel and was positioned on the crest of standard spillway and finished with sharp-edged. The water level from upstream to downstream was measured at the centerline of flume with a point gauge and having an accuracy of ±0.1 mm. The crest pressure was measured by static tube that was a piezometer board with glass tubes vented to the atmosphere. Measurements on the piezometer board were readable within of ±1 mm. The gate opening was measured accurately with a meter fixed on the gate and the accuracy of ±1 mm. The discharge in the flume was also measured by a rectangular sharp-crested weir located in gathering tank. The relative uncertainty in discharged measurement was about 3%. The details of experimental scheme could be seen in Fig. 3.

3.2 Experimental procedure

The experiments were carried out for various gate opening $G_o$, and water head $H_s$. The water surface profile, the crest pressures, the discharge was measured carefully for each gate opening case in steady-state flow. To prevent the effect of tailwater to the upstream of the spillway, the tailwater was kept below the critical depth and had no influence on the flow through the gated spillway for all test cases. The characteristics of the assembled data were summarized in Table 1 and some remain variable parameters in this study were also shown in Fig. 2 and Fig. 3.

**Table 1. Range of variables studied**

<table>
<thead>
<tr>
<th>No. of experiments</th>
<th>$Q$ (m³/s)</th>
<th>$G_o$ (cm)</th>
<th>$H_s$ (cm)</th>
<th>Reynolds number</th>
<th>Froude number</th>
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<td>10.27-32.42</td>
<td>5.27e+4-1.92e+5</td>
<td>1.02-2</td>
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</table>

![Figure 3. The experimental hydraulic model scheme](image)

4. Results and Discussion

4.1 The comparison between equations in the first and second group

To make the analysis of results easier, these formulas were named in the following way, such as Ansar, Hager, HEC and Wortman method. The discharge results were used Eq. (4) to (9) to calculate which corresponds to each above method and compared to measured discharge. In order to quantify the differences, using the mean absolute percentage error (MAPE) [14] to compute the residual between the estimated ($Q_{est}$) and measured discharge ($Q_{meas}$) for N point. This value is an error based on characteristic index that indicates the accuracy of a method in relative units. MAPE value is defined as follows:

$$MAPE = 100 \frac{1}{N} \frac{\sum\left(Q_{meas}[i] - Q_{est}[i]\right)}{Q_{meas}[i]}$$

(10)

in which, $N$ is the number of data, $Q_{est}[i]$ and $Q_{meas}[i]$ are the calculated and experimental value of discharge, respectively. According to Sepúlveda [10], the lower MAPE value, the more accuracy in each method and vice versa. Besides, the relative error can be calculated for particular case as follows:

$$\varepsilon_r = \frac{Q_{meas}[i] - Q_{est}[i]}{Q_{meas}[i]} \times 100\%$$

(11)

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The Fig. 4 showed the results of MAPE for Hager, HEC, Wortman and Ansar methods. These values are 4.295%, 3.83%, 3.281% and 2.975% respectively, which had a good estimation base on above definition. All of them are less than 5% which are acceptable in engineering practice. According to the definition of MAPE criteria, the approach of Ansar method achieves the most accuracy to state flow through the vertical gated spillway, in comparison with other methods. It means that the assumption for calculation $Z_v$ value of second group might be a little reasonable than first group. Additionally, in Ansar method, the discharge coefficient takes constant 0.75 that is not function of hydraulic parameters as the Hager or Wortman method.

Furthermore, the relative error is also considered in each case. There was a remarkable fluctuation in these methods. The relative errors of Hager, HEC, Wortman and Ansar methods were in a range (-5.64% ~ 8.42%), (-12.24% ~ 5.7%), (-8.69% ~ 7.39%) and (-0.54% ~ -11.04%), respectively. Although the value of MAPE in each method was small enough but the oscillation of $r$ was quite large. The highest value fluctuates up to 12.24% in HEC method and the lowest 0.54% in Ansar method. These values are quite larger in comparison with relative uncertainty of discharge measurement. The reason for such a large deviation was the selection of discharge coefficient $C_{o_p}$ value for each method. For example, HEC method selected the $C_{o_p}$ in a range of (0.6 ~ 0.8). It is not easy to choose exactly because the range is quite large. In this method, the relative error range was calculated with $C_{o_p} = 0.65$. If the $C_{o_p}$ value was chosen equal to 0.7, so, the relative error increased in the range of (-1.55% ~ -20.87%). It was of great interest to see that all of above method could be applied in hydraulic engineering and the accuracy is acceptable and reliable. However, the discharge coefficient must be selected carefully to minimize the error in specific cases.

### 4.2 A new formula for discharge coefficient $C_{o_p}$

To improve precision and simplify the calculation, some assumptions are often applied to determine the vertical pressure distribution $p(z)$ when the downstream profile of spillway is curved such as WES, Creager, or parabola profiles [2]... Respecting the Eq. (2) & (3), these formulas need to be made some assumptions (e.g., the pressure distribution, the vena-contraction coefficient) to calculate the discharge capacity of the flow in this field. In this work, the distribution of $p(z)$ at the section C-C was supposed as hydrostatic pressure and the $C_{o_p}$ value is taken approximate as unity. As a consequence, after integration Eq. (3), value of $Z_v$ could be calculated by equation as follows:

$$Z_v = H_o - G_o$$  (12)

The Eq. (2) is rewritten as below:

$$Q_{o_p} = C_{o_p} G_b \sqrt{2g(H_o - G_o)}$$  (13)

Based on this assumption, the effective hydraulic parameters on the discharge coefficient $C_{o_p}$ with the free flow condition is a function written as follows:

$$C_{o_p} = \Psi \left( v, H_o, G_o, P, H_o, g, \mu, \rho \right)$$  (14)

After dimensional analysis, non-dimensional parameters can be written as:

$$C_{o_p} = \Psi \left( Re, Fr, \frac{G_o}{H_o}, \frac{H_d}{H_o}, \frac{P}{H_o} \right)$$  (15)

where $Re$ and $Fr$ are dimensionless parameters Reynolds and Froude number regarding to gate opening, respectively. According to Table 1, one can see that the Re number is larger than $10^6$ so the viscous effect become less and less important. It means that the effect of viscous may be neglected [15]. Besides, the effect of ratio $H/P_o$ could be ignored. For that reason, $C_{o_p}$ was only a function of the other parameters except $Re$ number and $H/P_o$. We can specify $\Psi$ by performing experimental studies.
Table 2. Results of experimental discharge (Q) in this study and comparison with other methods
(Q_{a} = Hager, 1998; Q_{hec} = Hydrologic Center, 2002; Q_{c} = Ansar, 2001; Q_{w} = Wortman, 1989;
Q_{pre} is calculated by using Eq. (12) and Eq. (19))

<table>
<thead>
<tr>
<th>No.</th>
<th>G/H₀</th>
<th>H₀ (m³/s)</th>
<th>Q_{hec} (m³/s)</th>
<th>Q_{a} (m³/s)</th>
<th>Q_{w} (m³/s)</th>
<th>Q_{pre} (m³/s)</th>
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From the measure value of Q_{mean}H₀ in each specific experiment case, the estimated discharge coefficient
C_{pren} was calculated by using formula C_{pren} = Q_{mean} / [G_{a}L \sqrt{2g(H_{o} - G_{o})}] for all cases. The relative uncertainty in
calculating C_{pren} values was found to be approximately 3.1%. To validate the effect of parameters for this value,
linear and nonlinear multiple regression techniques [16] were used to determine Ψ function. The variations of the
independent parameters G_{a}/H₀, Fr_{g,a}, H₀/Hₚ with C_{pren} are illustrated through Figs. 5a-c. As shown in Fig. 5a, C_{pren}
shows a negative correlation with Fr_{g,a} indicating a decrease in C_{pren} with an increase in Fr_{g,a}. Fig. 5b and 5c show
positive correlations with G_{a}/H₀ and H₀/Hₚ indicating an increase in C_{pren} with an increase in G_{a}/H₀ and H₀/Hₚ with
polynomial and power variation.

In this proposal, C_{pren} can be written by multi linear regression (MLR) and nonlinear regression (NLR) equation as follows:
\[ C_{dpv} = \alpha_1 + \alpha_2 \left( \frac{G_o}{H_o} \right) + \alpha_3 \left( \frac{H_o}{F_r} \right) \]
\[ C_{dpv} = \alpha_1 + \alpha_2 \left( \frac{G_o}{H_o} \right)^{\beta_1} + \alpha_3 \left( \frac{H_o}{F_r} \right)^{\beta_2} + \alpha_4 \left( \frac{H_o}{F_r} \right)^{\beta_3} \]

where \( \alpha_1, \alpha_2, \ldots, \beta_1, \beta_2, \ldots \) are constant unknown parameters, which can be found by using the experimental data. Additionally, it can be seen from Fig. 5 that \( H_o / F_r \) is less important than the other parameters, so this ratio can be omitted from regressing procedure to simplifying equations. Using the ordinary least square method, the following equation was obtained in this case:

\[ C_{dpv} = 0.0294 + 1.0671 \left( \frac{G_o}{H_o} \right) + 0.2621 \left( F_r \right), R^2 \approx 0.975 \] (18)

\[ C_{dpv} = 1.1398 + 1.645 \left( \frac{G_o}{H_o} \right)^{1.5831} - 1.0986 \left( F_r \right)^{0.9362}, R^2 = 0.996 \] (19)

These formulas show that \( C_{dpv} \) value has just influenced by the ratio \( G_o / H_o \) and \( F_r \) number of the gate opening. Moreover, all of unknown coefficient is quite meaningful with significance level 5% that are very important in statistic theory. So, it is recommended to use Eq. (18) and Eq. (19) due to their simplicity and ease of using. Besides, nonlinear equation has more precision compared to linear form, so it is better to use this nonlinear equation.

**Figure 5a.** \( C_{dpv} \) versus \( F_r \)

**Figure 5b.** \( C_{dpv} \) versus \( G_o / H_o \)

**Figure 5c.** \( C_{dpv} \) versus \( H_o / H_o \)

**Figure 6.** Measured versus estimated discharge using Eq. (12) and Eq. (19)

Using Eq. (12) and Eq. (19) to calculate \( Z_o \) and \( C_{dpv} \), and the estimated discharge \( Q_{pv} \) was computed by Eq. (13). As a result, the MAPE value of \( Q_{pv} \) was calculated for comparison purpose to the other methods. According to Fig. 4, this value was the smallest 0.351% as compared to the other methods and having relative error in range (-0.76% to 1.11%). It means that the present method has a more accuracy than the other methods within the limitations of this study as follows: \( 1.02 < F_r < 2.0 \), \( 0.24 < G_o / H_o < 0.65 \). The correlation coefficient
between estimated and measured discharge is presents in Fig. 6. As shown in this figure, there is a good correlation between measured and estimated discharge values and approximate unity. This result is quite reasonable and reliable. This proves that the assumption of hydrostatic pressure distribution at section C-C and vertical contraction is approximate unity are acceptable in hydraulic practice, although it is not entirely accurate.

5. Conclusions

This paper has conducted a comparison among discharged equations of gated spillway by using experimental data of a laboratory flume. The accuracy of each method has been examined and showing that the MAPE value in the range of ±5%. This value was to ensure the reliability of these methods. However, these methods also show that the relative error has a large deviation because the choice of the exact discharge coefficient in each equation is not quite simple. To improve the accuracy, alternatively, we introduced a new discharge coefficient \( C_w \) equation that based on nonlinear multiple regression and ordinary least squares method from experimental data. It is the function of the Froude number, ratio of gate opening to headwater depth. The result shows that there is a good agreement between estimated discharge as using Eq. (12) and Eq. (19) with measured data in limited conditions of laboratory as follows: \( 1.02 < Fr < 2 \), \( 0.24 < G_r / H < 0.65 \). In short, these methods, which are introduced in this paper, are accurate enough to estimate discharge through the gated standard spillway. However, the selection of \( C_w \) value needs to be careful in calculation. Additionally, there should be further research to expand the scope of calculation, as well as studies determined the pressure distribution at outlet section (section C-C) in case the downstream profile is arbitrary curve. This result will improve the accuracy of discharged estimation for outlet works that is very important from a controlling flood, management or irrigation in developing water resources.

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