PROBABILITY PUSHOVER ANALYSIS OF REINFORCED CONCRETE FRAME STRUCTURES USING DROPOUT NEURAL NETWORK

Dang Viet Hung\textsuperscript{a,*}, Nguyen Truong Thang\textsuperscript{a}, Pham Xuan Dat\textsuperscript{a}

\textsuperscript{a}Faculty of Building and Industrial Construction, National University of Civil Engineering, 55 Giai Phong road, Hai Ba Trung district, Hanoi, Vietnam

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\section*{Abstract}
When taking into consideration nonlinear phenomena such as material plasticity, plastic hinge, and P-Delta effect, the pushover analysis can provide more realistic structures' nonlinear responses. However, this method is not widely used in practice as it is more complex and requires more expertise than elastic approaches. On the other hand, the data-driven method emerges as an increasingly appealing alternative since it requires only input parameters, then directly yields results in conditions that enough training data are provided, as well as an appropriate machine learning model is devised. Thus, this study develops a probabilistic data-driven approach using the Multiple Layer Perceptron network coupled with the Dropout mechanism to perform the pushover analysis of reinforced concrete (RC) frame structures, predicting base shear, lateral displacement, as well as their relationship between the two formers. Moreover, corresponding confidence intervals of predicted values are also available owing to the probabilistic nature of the method, thus helping engineers design conservative solutions.

\textbf{Keywords:} pushover analysis; reinforced concrete; structure; probabilistic analysis; machine learning; dropout mechanism; OpenSees.

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\section*{1. Introduction}

In practice, assessment of lateral displacement of structures is of great importance to ensure its safety as well as the comfortability of users. Pushover analysis, a nonlinear static technique, is one of the most well-known methods to perform this task. However, though it can provide more accurate results, the nonlinear structural analysis is still less favorable compared to the linear counterpart due to its complexity, high computational cost and expertise required. In recent years, data-driven method has been considered as a promising alternative by which users only feed inputs to a properly trained model to obtain reliable results. A number of data-driven models have been developed in the field of civil engineering [1, 2]. However, when structures become more complex or the nonlinearity is more pronounced, a deterministic approach seems to be inappropriate because the uncertainty or the confidence of calculated results needs to be estimated. That is why probabilistic data-driven models gain increasing attention among the engineering community.

\textsuperscript{*}Corresponding author. \textit{E-mail address:} hungdv@nuce.edu.vn (Hung, D. V.)
For the probabilistic collapse analysis of structures due to vehicle impact, an artificial neural network, which is a surrogate model efficient in computational time, was validated by comparison with experience as presented in [3]. It is also noticed that the surrogate model facilitates various types of sensitivity analysis, providing more insights into the effects of input parameters on the structure’s behavior. To assess the damage severity of building after an earthquake, i.e., from minor damage to collapsed structures, Mangalathu proposed a deep learning-based approach trained on a dataset of 3423 buildings recorded after the 2014 South Napa earthquake [4]. The method achieved a highly accurate result of 86% and could be integrated into a mobile application, thus enabling a broad range of users to estimate the vulnerability of the structure immediately following seismic events. Besides, Feng et al. proposed a probabilistic framework dealing with the progressive collapse of reinforced concrete (RC) structures due to column removal based on the probability density evolution method [5]. Such a method allows for quantifying uncertainty in the quantities of interest, i.e., resisting forces, displacement, etc. with statistic features such as mean variation, standard deviation and probability density function. Brunesi et al. [6] carried out both deterministic analysis and probabilistic analysis of progressive collapse in low-rise RC buildings, through which the authors emphasized the contribution of secondary structural elements such as framing beam in progressive collapse resistance by providing additional alternative load path. Guo et al. [7] conducted a series of Monte Carlo simulation with the Latin hypercube sampling method on a 1-h rated steel beam and proved that the beam is able to resist natural fire event with a probability of failure under 10%. Although Monte Carlo-based method is able to perform probabilistic analysis but due to its computational expensiveness, another more practical probabilistic framework for industry application was suggested to be explored. Worrell et al. [8] investigated a machine learning-based (ML) model for probabilistic assessment of the safety of nuclear power plants against fire hazard, specifically various ML models are implemented and compared together, such as K-nearest neighbor, support vector machine, Decision Tree regressor, and algebraic fire models. It is noticed that these ML models require a large number of training data to reach a satisfying prediction accuracy. Recently, Fengfu investigated the progressive collapse of steel frame under fire by leveraging both ML approaches and Monte Carlo simulations [9]. The obtained results are promising, though it requires sufficient large dataset for the training process which hinder its applicability in real-world applications.

In an effort to extend the probabilistic data-driven method to estimate the lateral displacement of building structures, this study develops a probabilistic data-driven approach using the Multiple Layer Perceptron network coupled with the Dropout mechanism to perform the pushover analysis of RC frame structures. Once trained, the method can provide quantities of interest such as base shear, lateral displacement, as well as their relationship between the two formers without requiring building a numerical model such as Finite Element Method (FEM) with much reduced computational time. Moreover, the corresponding confidence intervals of predicted values are also available owing to the probabilistic nature of the method, thus helping engineers design conservative solutions.

2. Dropout Neural Networks

In this section, the architecture of Dropout Neural Networks inspired from the work of Gal and Ghahramani [10] is investigated and adapted for the progressive collapse problem. The idea behind the use of Dropout mechanism in this work is that such type of neural network (NN) allows effectively handling the scarcity of collapsed data by providing probabilistic distribution and its statistic moments rather than point estimates. Dropout Network consists of three key aspects: (i) Multiple layers of perceptron includes input layers, one or more hidden layers and output layer as in the conventional
Deep Neural Networks (DNN); (ii) Dropout mechanism is applied to the hidden layers and the input layer with various ratios, whose optimal values are obtained from fine-tuning processes; and (iii) The weight of whole neural network is shared among thinned neural networks resulting from Dropout. The architecture of the NN with Dropout mechanism is schematically illustrated in Fig. 1.

![Architecture of Neural Network with Dropout mechanism](image)

Figure 1. Architecture of Neural Network with Dropout mechanism
(a) The conventional plain neural network where every perceptron in a layer are connected to all perceptrons of the next layer; (b) The NN with Dropout where a fraction of number of perceptron (circled cross) is randomly neglected during calculation

2.1. Deep Neural Networks (DNN)

Conceptually, DNN mimic the information analysis process of the brain, in which meaningful sets of information are extracted from received data through a complex system of neurons connected together. Hence, a typical DNN architecture consists of multiple layers, each layer comprised various number of perceptrons, together they form a nonlinear mapping from input data to output results. Fig. 1(a) depicts an example of the DNN architecture. Mathematically, one can formulate this nonlinear mapping as follows [11]:

$$Y = F(X|W) = f_L(...(f_2(f_1(X|W_1)|W_2)...W_L)$$

where $L$ is the total number of layers in the network including the input and output layers, $W$ is the matrix of the network’s parameters; $X$ and $Y$ denote the input and output vectors, respectively.

The connection between two consecutive layers is performed by combining a linear matrix operation and a nonlinear activation function as follows:

$$f_l(X_l|W_l) = h(W_l \times X_l + b_l) \text{ with } l = 1, 2, \ldots, L$$

where $X_l$ is the input of the $l$ layer; $b_l$ is bias vector; and $h$ denotes the nonlinear activation function. In this study, one adopts the Rectified Linear Unit (ReLU) function for hidden layers and the softmax function for the output layers.

Herein, the network parameters are computed in a supervised fashion, which means the structural state is labeled in advance, then the database is divided into three groups, namely training, validation,
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and testing datasets. The values of network parameters are randomly initialized, then they are iteratively updated via the backpropagation algorithm to minimize the deviation denoted by a loss function $L$ between prediction values and annotated structural states. At a step $t$ of the training process, the parameter values are updated by:

$$w_{ij}^l(t) = w_{ij}^l(t-1) + \eta \delta_i^l x_{i-1}^l(t)$$

(3)

where $l$ denotes the layer number; $\delta_i^l$ is the error at node $i$ of layer $l$, which is calculated backward from the derivation at output layer $\delta_L$; and $\eta$ stands for the learning rate. The training process terminates when a tolerable error level is met or the number of iterations reaches a limit.

2.2. Dropout mechanism

In general, deep neural networks contain a large number of parameters, i.e. weight matrix $W$, thus training such models requires a significant amount of available and well-curated data. If data is limited, these are many possible configurations which can provide accurate results on training data, but most of these formers eventually perform poorly on unseen testing data, namely the overfitting phenomenon. To tackle these difficulties, one can adopt a combination approach averaging prediction results of different configurations. Because a combined model is always more generalized than a single one, it will improve the prediction performance. However, it also required significantly increasing resources both in time and in efforts. More recently, Srivastava et al. [12] proposed the dropout mechanism which efficiently deals with limited data by providing a probability distribution rather than a deterministic estimate. Then, the statistical properties such as the mean value and the standard deviation of quantities of interest can be derived. In this way, one is able not only to obtain a predicted result but also to assess how much the result confidence is. The mechanism of the dropout technique is described in detail as follows.

Considering a node $j$ in layer $l$, its state is independently sampled from a Bernoulli distribution with a predefined probability $p$, as follows:

$$r_{ij}^l \sim \text{Bernoulli}(p)$$

(4)

where $r_{ij}^l$ is a binary variable indicating the state of the considered node with 0 corresponding to be dropped out, while 1 to be active. Then the input vector for layer $l$ of the network is updated by:

$$\tilde{X}_l = X_l \odot r_l$$

(5)

where $\odot$ denotes the element-wise product, $X_l$ are input values for layer $l$ of the original plain network, $\tilde{X}_l$ for updated dropout network. Then, Eq. (2) is rewritten as below,

$$f_l(X_l|W_l) = h(W_l \times \tilde{X}_l + b_l)$$

(6)

The above process is repeated for every layer in the network architecture except the output layer. After that, the network is trained by performing a number of iteration of forward-pass for calculating output results and backward-pass to update the weight values $W$. Note that, at each iteration, different nodes are eliminated with a probability $p$, forming a thinner architecture than the original plain architecture.

Intuitively, the Dropout Network architecture can potentially improve the prediction ability thanks to two-folds: i) it works as a regularization technique reducing the overfitting problem by not being
over-relied on any node in the network structure, as it can be eliminated. ii) It is well-known that for a specific database, there is a particular best-fitted architecture providing the highest prediction accuracy, but it is not generalizable for other data. In contrast, the Dropout mechanism provides a more generalized architecture but not necessarily larger as the effect of eliminating nodes, which can provide non-optimal but satisfied prediction accuracy with different data.

3. Pushover analysis database

Pushover analysis (PA) is a widely used static procedure to approximate non-linear building lateral deformations beyond the elastic range, up to the failure occurrence. Compared to elastic analysis, PA can encompass more realistic behaviors including plastic hinges, non-linear constitutive law of material, P-Delta effect, and staged construction depending user-defined tasks. In this study, to perform PA of structures, one adopts the open-source program OpenSees [13] credited by the Pacific Earthquake Engineering Research Center thanks to its accurateness, openness, and high computation speed, appreciated by the earthquake research community.

![Figure 2. Typical RC frame under a concentrated horizontal force at the top floor for pushover analysis](image)

In this section, an example using OpenSees to carry out a PA is briefly demonstrated. Let’s take an example of a 2-D asymmetric 3-story and 2-bay RC frame with a constant story height \( H \), and its bay widths \( B \) as shown in Fig. 2. The frame is subjected to uniformly distributed vertical load acting on all its beams in addition to the self-weight. A concentrated horizontal force is applied at the leftmost top floor.

The nonlinear constitutive law of steel is constructed using the model of Filippou et al. [14]. At first, the behavior of the steel is approximated by a bilinear curve composed of a branch for an initial linear elastic state with slope \( E_0 \), and another branch for hardening state characterized by a ratio \( b \) resulting in a slope \( E_1 = bE_0 \). Next, the transition between the two states is controlled by a curvature parameter \( R_0 \), as shown in Fig. 3(a). The values of \( b \) and \( R_0 \) are initially adopted as 0.02 and 20 as recommendations in general cases by [13].

The constitutive law of concrete is simulated using the Kent-Scott-Park model [15] composed of three parts, as shown in Fig. 3(b) where negative and positive values denote compression and tension, respectively. The elastic state of compression starts with an initial slope \( E_0 \), reaching the maximum strength of \( f_{pc} \), i.e., 28-day compressive strength, and the corresponding strain \( \varepsilon_0 \). Mathematically, \( E_0 \) can be calculated by the following equation:

\[
E_0 = \frac{2f_{pc}}{\varepsilon_0} \tag{7}
\]
After reaching $f_{pc}$, the strength decreases, while the compressive strain continuously increases up to $\varepsilon_u$, so-called crushing strain, and respective crushing strength $f_{pcU} = 0.2f_{pc}$. On the other hand, the tensile behavior of concrete is modeled by a bilinear curve of which the initial branch has the same slope $E_0$, after reaching the maximum tensile strength $f_t$, the tensile strength decreases with a softening slope $E_{ts}$.

In terms of section modeling, the section of RC element is simulated using the fiber approach which is able to account for moment-curvature, axial force-deformation, and their interaction at the same time [16] thus being superior than the uniaxial section approach calculating independently the bending and normal stress. Specifically, each concrete section is decomposed into a predefined number of fibers, horizontally and vertically as shown in Fig. 4. For the rebar, the section is defined with either steel evenly distributed around the perimeter as in columns or per-layer (top/intermediate/bottom) as in beams.

The output of interest of the simulation is the pair $(D_{top}, F_{base})$, which are the drift ratio of the top floor and the corresponding base shear. Fig. 5 depicts a typical relationship between $D_{top}$ and $F_{base}$ during PA. It can be seen that with a small drift ratio ($< 0.01$), the base shear linearly increases with increasing drift, after passing an yield point, a likely plateau region occurs before a softening trend drives the structure’s behavior.
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**Figure 4.** Fiber section modelling

In terms of section modelling, the section of RC element is simulated using the fiber approach which is able to account for moment-curvature, axial force-deformation, and their interaction at the same time [16] thus being superior than the uniaxial section approach calculating independently the bending and normal stress. Specifically, each concrete section is decomposed into a predefined number of fibers, horizontally and vertically as shown in Figure 4. For the rebar, the section is defined with either steel evenly distributed around the perimeter as in columns or per-layer (top/intermediate/bottom) as in beams.

**Figure 5.** Typical force-displacement curve obtained from PA using FEM.

The output of interest of the simulation is the pair \((A,B)\), which are the drift ratio of the top floor and the corresponding base shear. Figure 5 depicts a typical relationship between \(A\) and \(B\) during PA. It can be seen that with a small drift ratio (<0.01), the base shear linearly increases with increasing drift, after passing an

**Table 1.** Pushover database for training the Dropout network obtained from FEM

| No | \(N_s\) | \(N_{bay}\) | \(H_s\) (m) | \(L_{bay}\) (m) | \(H_{col}\) (m) | \(H_{beam}\) (m) | \(B_{beam}\) (m) | \(E\) (GPa) | \(f_y\) (MPa) | \(f_c\) (MPa) | \(\varepsilon_0\) (%) | \(\varepsilon_f\) (%) | \(\varepsilon_u\) (%) | \(\Delta\) % | \(F_{base}\) kN |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 3  | 3  | 2  | 3  | 0.3 | 0.3 | 0.4 | 180 | 300 | 20  | 0.2 | 2   | 0.02 | 8   | 0.5 | 300 |
| 2  | 3  | 3  | 2  | 3  | 0.3 | 0.3 | 0.4 | 180 | 300 | 20  | 0.2 | 2   | 0.02 | 8   | 1   | 340 |
| 3  | 3  | 3  | 2  | 3  | 0.3 | 0.3 | 0.4 | 180 | 300 | 20  | 0.2 | 2   | 0.02 | 8   | 1.5 | 380 |
| .  | .  | .  | .  | .  | .   | .   | .   | .   | .   | .   | .   | .   | .   | .   | .   | .   |
| Min| 1  | 1  | 2  | 2  | 0.3 | 0.3 | 0.3 | 180 | 300 | 20  | 0.2 | 2   | 0.02 | 8   | 0.5 | 4   |
| Max| 10 | 6  | 6  | 6  | 0.6 | 0.6 | 0.6 | 220 | 400 | 40  | 0.4 | 5   | 0.03 | 12  | 5   | 717 |

4. **Computational results**

In this section, the training process and test results of the Dropout network are presented. For the architecture of the network, a preliminary study conducted by the authors pointed out that an architecture of 15/256/128/64/32/1, meaning 5 layers: input layer with 15 perceptrons for 15 variables and 4 hidden layers with the numbers of perceptrons are 256, 128, 64 and 32, respectively and output layer is 1 corresponding to the base shear value. More detail of the grid search for selecting an appropriate architecture of MLP can be found in [1]. For the value of Dropout probability, one adopts the value

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0.5 recommended by [12]. In terms of training process hyper-parameters, one used the Adam optimizer with an initial learning rate \( l = 0.001 \), which was reduced by a factor of 0.01 when there was no validation loss improvement for five consecutive iterations, and the total number of iterations is 300. The performance criteria, a.k.a, the loss function is defined using the mean square error (MSE) measuring the average of the squared difference between the predicted base shears and actual values as follows:

\[
MSE = \frac{1}{N} \sum_{i=1}^{N} \left( F_{\text{actual base,}i} - F_{\text{predict base,}i} \right)^2
\]

(8)

The evolution of the mean square error (MSE) between base shears predicted by the Dropout network and those from the FEM model during the training process is depicted in Fig. 6. As expected, the MSE decreases with increasing numbers of iterations; in addition, the histogram of base shears predicted are also plotted at the bottom, visually show the improvement of the model’s performance. At the end of the training process, an excellent agreement between results is apparently observed.

After the training process, the Dropout network is validated using experimental PA results reported in [19]. The experiment consists of a 2-story and 2-bay RC frame, as shown in Fig. 7 with detailed dimensions, whose columns’ cross-sections is 400 \( \times \) 400 mm\(^2\) with 10-D19 rebar and beams’ cross-section is 300 \( \times \) 400 mm\(^2\) with 8-D19 rebar, respectively.

The comparison results are illustrated in Fig. 8, showing a very good agreement between the experimental curve and the prediction curve from the present method. In addition, corresponding 90% confidence interval (CI), represented by shaded area is also derived by repeating the inference 100 times, each time the architecture of the network is stochastically altered by the Dropout mechanism. As is known, 90% CI means there is a 90% probability that the actual value of concerned quantity lies within the CI.

Note that the elastic period has relatively small CI than the plastic zone, i.e., less uncertainty, and the CI is widened as the structure is more damaged. In short, this comparison result reliably confirms the accuracy of the present Dropout network.
Next, the Dropout network is used to perform parametric studies, providing more insights about the influences of main input parameters on structures’ non-linear behavior. Starting from the above two-bay RC frame, one varies the story height, the number of stories, the column size, and the bay width, then calculates the base shear corresponding to 5% drift ratio, and plots evolution curves. For clarity, only one parameter is modified each time, while the other inputs remain the same as the experimental model.

![Experimental RC frame](image)

Figure 7. Experimental RC frame [19]

Fig. 8 demonstrates the evolution of base shear at a 5% drift ratio in the functions of these investigated parameters. It can be seen that the base shear decreases with increasing story height, though a greater story height meaning a larger lateral displacement. This implied that the columns lost their strength with such large displacement. A decreasing trend of base shear is also observed as the number of stories increases, but with a significantly slow rate owing to the story beams contributing to the frame’s lateral rigidity and reduce columns’ slenderness. In contrast, increasing column size strengthen the frame rigidity; thus, the base shear manifests an upward tendency. Meanwhile, the bay width interestingly shows two regimes: an upward trend from 2 to 4 m, then a gradual downward trend. Similar parametric studies for other parameters or even multiple parameters can be carried out with the help of the Dropout network depending on users’ interests.

![Base shear at 5% drift ratio](image)

Figure 8. Base shear at 5% drift ratio in the functions of input parameters
5. Conclusions

In this study, a data-driven approach for pushover analysis of RC frames is investigated by leveraging the probabilistic nature of the Dropout mechanism and the well-known nonlinear mapping capacity of the neural network, providing a novelty alternative to the conventional model-based methods. Knowing that training data is of paramount importance for any data-driven approach, a rigorous numerical simulation suite has been carried out using the reliable OpenSees software accredited by the engineering community accounting for the nonlinearity of both concrete and steel materials, as well as the forming of plastic hinges. The simulation suite results in a 10000-sample database sufficiently large for training a data-driven model.

After being trained, the Dropout network is validated through a comparison with experimental measurement published in the literature, which confirms its accuracy and also displays the respective confidence interval, which offers a margin of safety for design solutions. Next, parametric studies are conducted with the help of the present method whose results reveal more insight into the structures’ behavior, such as the relationship between the number of stories and the base shear, the effect of column size, and material properties in reducing the base shear. For the type of structure considered in this study, one posits that there are a positive correlation between the lateral rigidity of the structure with column sizes, a negative correlation with the story height, and a bimodal positive-then-negative correlation with the bay width.

In the future, it is interesting to extend the method to structures’ behavior under cyclic loads or even under extreme loading as ground motions, shock loading, but a more sophisticated neural network architecture will be resorted. Another promising direction is to account for multiple criteria rather than a single pair \( (D_{\text{top}}, F_{\text{base}}) \) as done in the real design process of structures for which an ensemble of dropout models will be developed; each model specifically solves a single criterion.

References

Appendix A.

As a second validation case, one compares the Dropout network with a FEM unseen on training data in performing pushover analysis. The structure of interest has 9 stories with input parameters are listed in Table A.1. The figure show that the predicted results encompasses those from FEM, thus confirming the credibility of the proposed approach.

Table A.1. Input parameters for validation case with FEM

<table>
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<th>No</th>
<th>$N_s$</th>
<th>$N_{bay}$</th>
<th>$H_s$ (m)</th>
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<th>$H_{col}$ (m)</th>
<th>$H_{beam}$ (m)</th>
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Figure A.1. Validation results of Dropout network using FEM model.